APPLIED ECONOMETRICS FORMULAS

Simple Linear Regression

Let us define

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - n\bar{y}^2$$
$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - n\bar{x}\bar{y}$$

and

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$$

Then (3.8) can be written as

$$\hat{\beta} S_{xx} = S_{xy} \quad \text{or} \quad \hat{\beta} = \frac{S_{xy}}{S_{xx}} \tag{3.9}$$

Hence the least squares estimators for α and β are

$$\boxed{\hat{\beta} = \frac{S_{xy}}{S_{xx}} \quad \text{and} \quad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}} \tag{3.10}$$

The estimated residuals are

$$\hat{u}_i = y_i - \hat{\alpha} - \hat{\beta} x_i$$

The residual sum of squares (to be denoted by RSS) is given by

$$\begin{aligned} \text{RSS} &= \sum (y_i - \hat{\alpha} - \hat{\beta} x_i)^2 \\ &= \sum [y_i - \bar{y} - \hat{\beta}(x_i - \bar{x})]^2 \\ &= \sum (y_i - \bar{y})^2 + \hat{\beta}^2 \sum (x_i - \bar{x})^2 - 2\hat{\beta} \sum (y_i - \bar{y})(x_i - \bar{x}) \\ &= S_{yy} + \hat{\beta}^2 S_{xx} - 2\hat{\beta} S_{xy} \end{aligned}$$

But $\hat{\beta} = S_{xy}/S_{xx}$. Hence we have

$$\text{RSS} = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = S_{yy} - \hat{\beta} S_{xy}$$

S_{yy} is usually denoted by TSS (total sum of squares) and $\hat{\beta} S_{xy}$ is usually denoted by ESS (explained sum of squares). Thus

$$\underset{\text{(total)}}{\text{TSS}} = \underset{\text{(explained)}}{\text{ESS}} + \underset{\text{(residual)}}{\text{RSS}}$$

The coefficient of determination r_{xy}^2 is given by

$$r_{xy}^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{\text{TSS} - \text{RSS}}{\text{TSS}} = \frac{\hat{\beta} S_{xy}}{S_{yy}}$$

Summary

The estimates for the regression coefficients are

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} \quad \text{and} \quad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

The residual sum of squares is given by

$$\text{RSS} = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = S_{yy} - \hat{\beta} S_{xy} = S_{yy}(1 - r_{xy}^2)$$

and the coefficient of determination is given by

$$r_{xy}^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = \frac{\hat{\beta} S_{xy}}{S_{yy}}$$

The least squares estimators $\hat{\beta}$ and $\hat{\alpha}$ yield an estimated straight line that has a smaller RSS than any other straight line.

$\hat{\alpha}$ and $\hat{\beta}$ are jointly normally distributed with

$$E(\hat{\alpha}) = \alpha \qquad \text{var}(\hat{\alpha}) = \sigma^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right)$$

$$E(\hat{\beta}) = \beta \qquad \text{var}(\hat{\beta}) = \frac{\sigma^2}{S_{xx}}$$

and

$$\text{cov}(\hat{\alpha}, \hat{\beta}) = \sigma^2 \left(\frac{-\bar{x}}{S_{xx}} \right)$$

These results would be useful if the error variance σ^2 were known. Unfortunately, in practice, σ^2 is not known, and has to be estimated.

If RSS is the residual sum of squares, then

$$\hat{\sigma}^2 = \frac{\text{RSS}}{n - 2} \text{ is an unbiased estimator for } \sigma^2$$

Also

$$\frac{\text{RSS}}{\sigma^2} \text{ has a } \chi^2\text{-distribution with degrees of freedom } (n - 2)$$

$(\hat{\alpha} - \alpha)/\mathrm{SE}(\hat{\alpha})$ and $(\hat{\beta} - \beta)/\mathrm{SE}(\hat{\beta})$ each have a t-distribution with d.f. $(n - 2)$. These distributions can be used to get confidence intervals for α and β and to test hypotheses about α and β.

$\hat{\sigma}$ is usually called the *standard error of the regression*. It is denoted by SER (sometimes by SEE).

Table 3.3 Analysis of Variance for the Simple Regression Model

Source of Variation	*Sum of Squares*	*Degrees of Freedom*	*Mean Square*
x	ESS $= \hat{\beta} S_{xy}$	1	ESS/1
Residual	RSS $= S_{yy} - \hat{\beta} S_{xy}$	$n - 2$	RSS/$(n - 2)$
Total	TSS $= S_{yy}$	$n - 1$	

Multiple Linear Regression

where

$$\bar{y} = \frac{1}{n}\sum y_i \qquad \bar{x}_1 = \frac{1}{n}\sum x_{1i} \qquad \bar{x}_2 = \frac{1}{n}\sum x_{2i}$$

Equation (4.3) can be written as

$$\sum x_{1i}y_i = \hat{\alpha}\sum x_{1i} + \hat{\beta}_1 \sum x_{1i}^2 + \hat{\beta}_2 \sum x_{1i}x_{2i}$$

Substituting the value of $\hat{\alpha}$ from (4.5) into this equation, we get

$$\sum x_{1i}y_i = n\bar{x}_1(\bar{y} - \hat{\beta}_1\bar{x}_1 - \hat{\beta}_2\bar{x}_2) + \hat{\beta}_1 \sum x_{1i}^2 + \hat{\beta}_2 \sum x_{1i}x_{2i} \qquad (4.6)$$

We can simplify this equation by the use of the following notation. Let us define

$$S_{11} = \sum x_{1i}^2 - n\bar{x}_1^2 \qquad S_{1y} = \sum x_{1i}y_i - n\bar{x}_1\bar{y}$$
$$S_{12} = \sum x_{1i}x_{2i} - n\bar{x}_2\bar{x}_2 \qquad S_{2y} = \sum x_{2i}y_i - n\bar{x}_2\bar{y}$$
$$S_{22} = \sum x_{2i}^2 - n\bar{x}_2^2 \qquad S_{yy} = \sum y_i^2 - n\bar{y}^2$$

Equation (4.6) can be written as

$$S_{1y} = \hat{\beta}_1 S_{11} + \hat{\beta}_2 S_{12} \qquad (4.7)$$

By a similar simplification, equation (4.4) can be written as

$$S_{2y} = \hat{\beta}_1 S_{12} + \hat{\beta}_2 S_{22} \qquad (4.8)$$

Now we can solve these two equations to get $\hat{\beta}_1$ and $\hat{\beta}_2$. We get

$$\hat{\beta}_1 = \frac{S_{22}S_{1y} - S_{12}S_{2y}}{\Delta}$$

$$\hat{\beta}_2 = \frac{S_{11}S_{2y} - S_{12}S_{1y}}{\Delta} \qquad (4.9)$$

where $\Delta = S_{11}S_{22} - S_{12}^2$. Once we obtain $\hat{\beta}_1$ and $\hat{\beta}_2$ we can get $\hat{\alpha}$ from equation (4.5). We have

$$\hat{\alpha} = \bar{y} - \hat{\beta}_1\bar{x}_1 - \hat{\beta}_2\bar{x}_2$$

Thus the computational procedure is as follows:

1. Obtain all the means: $\bar{y}$, $\bar{x}_1$, $\bar{x}_2$.
2. Obtain all the sums of squares and sums of products: $\sum x_{1i}^2$, $\sum x_{2i}^2$, $\sum x_{1i}x_{2i}$, and so on.
3. Obtain S_{11}, S_{12}, S_{22}, S_{1y}, S_{2y}, and S_{yy}.
4. Solve equations (4.7) and (4.8) to get β_1 and $\hat{\beta}_2$.
5. Substitute these in (4.5) to get $\hat{\alpha}$.

In the case of simple regression we also defined the following:

$$\text{residual sum of squares} = S_{yy} - \hat{\beta}S_{xy}$$

$$\text{regression sum of squares} = \hat{\beta}S_{xy}$$

$$r_{xy}^2 = \frac{\hat{\beta}S_{xy}}{S_{yy}}$$

The analogous expressions in multiple regression are

$$\text{RSS} = S_{yy} - \hat{\beta}_1 S_{1y} - \hat{\beta}_2 S_{2y}$$

$$\text{regression sum of squares} = \hat{\beta}_1 S_{1y} + \hat{\beta}_2 S_{2y}$$

$$\text{regression sum of squares} = \hat{\beta}_1 S_{1y} + \hat{\beta}_2 S_{2y}$$

$$R^2_{y\cdot 12} = \frac{\hat{\beta}_1 S_{1y} + \hat{\beta}_2 S_{2y}}{S_{yy}}$$

1. $\hat{\alpha}$, $\hat{\beta}_1$, and $\hat{\beta}_2$ have normal distributions with means α, β_1, β_2, respectively.
2. If we denote the correlation coefficient between x_1 and x_2 by r_{12}, then

$$\text{var}(\hat{\beta}_1) = \frac{\sigma^2}{S_{11}(1 - r_{12}^2)}$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{S_{22}(1 - r_{12}^2)}$$

$$\text{cov}(\hat{\beta}_1, \hat{\beta}_2) = \frac{-\sigma^2 r_{12}^2}{S_{12}(1 - r_{12}^2)}$$

$$\text{var}(\hat{\alpha}) = \frac{\sigma^2}{n} + \bar{x}_1^2 \text{ var}(\hat{\beta}_1) + 2\bar{x}_1\bar{x}_2 \text{ cov}(\hat{\beta}_1, \hat{\beta}_2) + \bar{x}_2^2 \text{ var}(\hat{\beta}_2)$$

$$\text{cov}(\hat{\alpha}, \hat{\beta}_1) = -[\bar{x}_1 \text{ var}(\hat{\beta}_1) + \bar{x}_2 \text{ cov}(\hat{\beta}_1, \hat{\beta}_2)]$$

$$\text{cov}(\hat{\alpha}, \hat{\beta}_2) = -[\bar{x}_1 \text{ cov}(\hat{\beta}_1, \hat{\beta}_2) + \bar{x}_2 \text{ var}(\hat{\beta}_2)]$$

$$r_{12}^2 = \frac{S_{12}^2}{S_{11}S_{22}}$$

Analogous to the other results in the case of simple regression, we have the following results:

3. If RSS is the residual sum of squares then RSS/σ^2 has a χ^2-distribution with degrees of freedom $(n - 3)$. This result can be used to make confidence interval statements about σ^2.
4. If $\hat{\sigma}^2 = RSS/(n - 3)$, then $E(\hat{\sigma}^2) = \sigma^2$ or $\hat{\sigma}^2$ is an unbiased estimator for σ^2.
5. If we substitute $\hat{\sigma}^2$ for σ^2 in the expressions in result 2, we get the estimated variances and covariances. The square roots of the estimated variances are called the standard errors (to be denoted by SE). Then

$$\frac{\hat{\alpha} - \alpha}{SE(\hat{\alpha})} \qquad \frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_1)} \qquad \frac{\hat{\beta}_2 - \beta_2}{SES(\hat{\beta}_2)}$$

each has a t-distribution with degrees of freedom $(n - 3)$.

In addition to results 3 to 5, which have counterparts in the case of simple regression, we have one extra item in the case of multiple regression, that of confidence regions and joint tests for parameters. We have the following results.

6. $F = \frac{1}{2\hat{\sigma}^2}[S_{11}(\hat{\beta}_1 - \beta_1)^2 + 2S_{12}(\hat{\beta}_1 - \beta_1)(\hat{\beta}_2 - \beta_2) + S_{22}(\hat{\beta}_2 - \beta_2)^2]$ has an F-distribution with degrees of freedom 2 and $(n - 3)$. This result can be used to construct a *confidence region* for β_1 and β_2 together and to test β_1 and β_2 together.

$$F = \frac{R^2 / k}{(1 - R^2) / n - k - 1}$$

Let the estimated regression equation be

$$\hat{y} = \hat{\alpha} + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

Now consider the prediction of the value y_0 of y given values x_{10} of x_1, and x_{20} of x_2, respectively. These could be values at some future date.

Then we have

$$y_0 = \alpha + \beta_1 x_{10} + \beta_2 x_{20} + u_0$$

Consider

$$\hat{y}_0 = \hat{\alpha} + \hat{\beta}_1 x_{10} + \hat{\beta}_2 x_{20}$$

The prediction error is

$$\hat{y}_0 - y_0 = \hat{\alpha} - \alpha + (\hat{\beta}_1 - \beta_1)x_{10} + (\hat{\beta}_2 - \beta_2)x_{20} - u_0$$

Since $E(\hat{\alpha} - \alpha)$, $E(\hat{\beta}_1 - \beta_1)$, $E(\hat{\beta}_2 - \beta_2)$, and $E(u_0)$ are all equal to zero, we have $E(\hat{y}_0 - y_0) = 0$. Thus the predictor $\hat{y}_0$ is unbiased. Note that what we are saying is $E(\hat{y}_0) = E(y_0)$ (since both $\hat{y}_0$ and y_0 are random variables). The variance of the prediction error is

$$\sigma^2\left(1 + \frac{1}{n}\right) + (x_{10} - \bar{x}_1)^2 \operatorname{var}(\hat{\beta}_1) + 2(x_{10} - \bar{x}_1)(x_{20} - \bar{x}_2)\operatorname{cov}(\hat{\beta}_1, \hat{\beta}_2) + (x_{20} - \bar{x}_2)^2 \operatorname{var}(\hat{\beta}_2)$$

In the case of k explanatory variables, this is

$$\sigma^2\left(1 + \frac{1}{n}\right) + \sum_{i=1}^{k}\sum_{j=1}^{k}(x_{i0} - \bar{x}_i)(x_{j0} - \bar{x}_j)\operatorname{cov}(\hat{\beta}_i, \hat{\beta}_j)$$

We estimate σ^2 by $\text{RSS}/(n - 3)$ in the case of two explanatory variables and by $\text{RSS}/(n - k - 1)$ in the general case.

In Section 4.3, result 6, we discussed an F-test to test hypotheses about β_1 *and* β_2. An alternative expression for this test is defined by the statistic

$$F = \frac{(\text{RRSS} - \text{URSS})/r}{\text{URSS}/(n - k - 1)} \tag{4.13}$$

where URSS = unrestricted residual sum of squares
RRSS = restricted residual sum of squares obtained by imposing the restrictions of the hypothesis
r = number of restrictions imposed by the hypothesis

we have $\text{URSS} = S_{yy}(1 - R^2)$, and $\text{RRSS} = S_{yy}$. Hence the test is given by

$$F = \frac{[S_{yy} - S_{yy}(1 - R^2)]/k}{S_{yy}(1 - R^2)/(n - k - 1)} = \frac{R^2}{1 - R^2} \cdot \frac{n - k - 1}{k} \tag{4.14}$$

which has an F-distribution with degrees of freedom k and $(n - k - 1)$. What

Table 4.5 Analysis of Variance for the Multiple Regression Model

Source of Variation	*Sum of Squares, SS*	*Degrees of Freedom, d.f.*	*Mean Square, SS/d.f.*	*F*
Regression	R^2S_{yy}	k	$R^2S_{yy}/k = \text{MS}_1$	$F = \dfrac{\text{MS}_1}{\text{MS}_2}$
Residual	$(1 - R^2)S_{yy}$	$n - k - 1$	$\dfrac{(1 - R^2)S_{yy}}{n - k - 1} = \text{MS}_2$	
Total	S_{yy}	$n - 1$		

$$1 - \bar{R}^2 = \frac{n - 1}{n - k - 1}(1 - R^2) \tag{4.20}$$

The Analysis-of-Variance Test

Suppose that we have two independent sets of data with sample sizes n_1 and n_2, respectively. The regression equation is

$$y = \alpha_1 + \beta_{11}x_1 + \beta_{12}x_2 + \cdots + \beta_{1k}x_k + u \qquad \text{for the first set}$$

$$y = \alpha_2 + \beta_{21}x_1 + \beta_{22}x_2 + \cdots + \beta_{2k}x_k + u \qquad \text{for the second set}$$

For the β's the first subscript denotes the data set and the second subscript denotes the variable. A test for stability of the parameters between the populations that generated the two data sets is a test of the hypothesis:

$$H_0\text{: } \beta_{11} = \beta_{21},\ \beta_{12} = \beta_{22},\ \ldots,\ \beta_{1k} = \beta_{2k},\ \alpha_1 = \alpha_2$$

If this hypothesis is true, we can estimate a single equation for the data set obtained by pooling the two data sets.

The F-test we use is the F-test described in Secction 4.8 based on URSS and RRSS. To get the unrestricted residual sum of squares we estimate the regression model for each of the data sets separately. Define

RSS_1 = residual sum of squares for the first data set

RSS_2 = residual sum of squares for the second data set

$\dfrac{\text{RSS}_1}{\sigma^2}$ has a χ^2-distribution with d.f. $(n_1 - k - 1)$

$\dfrac{\text{RSS}_2}{\sigma^2}$ has a χ^2-distribution with d.f. $(n_2 - k - 1)$

Since the two data sets are independent $(RSS_1 + RSS_2)/\sigma^2$ has a χ^2 distribution with d.f. $(n_1 + n_2 - 2k - 2)$. We will denote $(RSS_1 + RSS_2)$ by URSS. The restricted residual sum of squares RRSS is obtained from the regression with the pooled data. (This imposes the restriction that the parameters are the same.) Thus $RRSS/\sigma^2$ has a χ^2-distribution with d.f. $= (n_1 + n_2) - k - 1$.

$$F = \frac{(RRSS - URSS)/(k + 1)}{URSS/(n_1 + n_2 - 2k - 2)} \tag{4.22}$$

which has an F-distribution with degrees of freedom $(k + 1)$ and $(n_1 + n_2 - 2k - 2)$. This test is derived in the appendix to this chapter.

$$F = \frac{(RSS - RSS_1)/n_2}{RSS_1/(n_1 - k - 1)} \tag{4.27}$$

which has an F-distribution with d.f. n_2 and $n_1 - k - 1$. Here

RSS = residual sum of squares from the regression based on $n_1 + n_2$ observations; this has $(n_1 + n_2) - (k + 1)$ d.f.

RSS_1 = residual sum of squares from the regression based on n_1 observations; this has $n_1 - k - 1$ d.f.

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-k-1}$$

6.2 Durbin–Watson Test

The simplest and most commonly used model is one where the errors u_t and u_{t-1} have a correlation ρ. For this model one can think of testing hypotheses about ρ on the basis of $\hat{\rho}$, the correlation between the least squares residuals $\hat{u}_t$ and $\hat{u}_{t-1}$. A commonly used statistic for this purpose (which is related to $\hat{\rho}$) is the Durbin–Watson (DW) statistic, which we will denote by d. It is defined as

$$d = \frac{\sum_{2}^{n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{1}^{n} \hat{u}_t^2}$$

where $\hat{u}_t$ is the estimated residual for period t. We can write d as

$$d = \frac{\sum \hat{u}_t^2 + \sum \hat{u}_{t-1}^2 - 2 \sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

Since $\sum \hat{u}_t^2$ and $\sum \hat{u}_{t-1}^2$ are approximately equal if the sample is large, we have $d \simeq 2(1 - \hat{\rho})$. If $\hat{\rho} = +1$, then $d = 0$, and if $\hat{\rho} = -1$, then $d = 4$. We have $d = 2$ if $\hat{\rho} = 0$. If d is close to 0 or 4, the residuals are highly correlated.

The sampling distribution of d depends on the values of the explanatory variables and hence Durbin and Watson[1] derived upper (d_U) limits and lower (d_L) limits for the significance levels for d. There are tables to test the hypothesis of zero autocorrelation against the hypothesis of first-order positive autocorrelation. (For negative autocorrelation we interchange d_L and d_U.)

If $d < d_L$, we reject the null hypothesis of no autocorrelation.
If $d > d_U$, we do not reject the null hypothesis.
If $d_L < d < d_U$, the test is inconclusive.

Hannan and Terrell[2] show that the upper bound of the DW statistic is a good approximation to its distribution when the regressors are slowly changing. They argue that economic time series are slowly changing and hence one can use d_U as the correct significance point.

The significance points in the DW tables at the end of the book are tabulated for testing $\rho = 0$ against $\rho > 0$. If $d > 2$ and we wish to test the hypothesis $\rho = 0$ against $\rho < 0$, we consider $4 - d$ and refer to the Durbin–Watson tables as if we are testing for positive autocorrelation.

Consider now the demand and supply model

$$\begin{aligned} q &= a_1 + b_1 p + c_1 y + u_1 \qquad \text{demand function} \\ q &= a_2 + b_2 p + c_2 R + u_2 \qquad \text{supply function} \end{aligned} \tag{9.2}$$

q is the quantity, p the price, y the income, R the rainfall, and u_1 and u_2 are the error terms. Here p and q are the endogenous variables and y and R are the exogenous variables. Since the exogenous variables are independent of the error terms u_1 and u_2 and satisfy the usual requirements for ordinary least squares estimation, we can estimate regressions of p and q on y and R by ordinary least squares, although we cannot estimate equations (9.2) by ordinary least squares. We will show presently that from these regressions of p and q on y and R we can recover the parameters in the original demand and supply equations (9.2). This method is called *indirect least squares*—it is indirect because we do not apply least squares to equations (9.2). The indirect least squares method does not always work, so we will first discuss the conditions under which it works and how the method can be simplified. To discuss this issue, we first have to clarify the concept of identification.

If we solve the two equations in (9.2) for q and p in terms of y and R, we get

$$\begin{aligned} q &= \frac{a_1 b_2 - a_2 b_1}{b_2 - b_1} + \frac{c_1 b_2}{b_2 - b_1} y - \frac{c_2 b_1}{b_2 - b_1} R + \text{an error} \\ p &= \frac{a_1 - a_2}{b_2 - b_1} + \frac{c_1}{b_2 - b_1} y - \frac{c_2}{b_2 - b_1} R + \text{an error} \end{aligned} \tag{9.3}$$

These equations are called the *reduced-form equations*. Equations (9.2) are called the *structural equations* because they describe the structure of the economic system. We can write equations (9.3) as

$$q = \pi_1 + \pi_2 y + \pi_3 R + v_1 \qquad (9.4)$$
$$p = \pi_4 + \pi_5 y + \pi_6 R + v_2$$

where v_1 and v_2 are error terms and

$$\pi_1 = \frac{a_1 b_2 - a_2 b_1}{b_2 - b_1}, \quad \pi_2 = \frac{c_1 b_2}{b_2 - b_1}, \quad \text{etc.}$$

The π's are called *reduced-form parameters*. The estimation of the equations (9.4) by ordinary least squares gives us consistent estimates of the reduced form paramters. From these we have to obtain consistent estimates of the parameters in equations (9.2). These parameters are called *structural parameters*. Comparing (9.3) with (9.4) we get

$$\hat{b}_1 = \frac{\hat{\pi}_3}{\hat{\pi}_6} \qquad \hat{b}_2 = \frac{\hat{\pi}_2}{\hat{\pi}_5},$$
$$\hat{c}_2 = \hat{\pi}_6(\hat{b}_1 - \hat{b}_2) \qquad \hat{c}_1 = -\hat{\pi}_5(\hat{b}_1 - \hat{b}_2)$$
$$\hat{a}_1 = \hat{\pi}_1 - \hat{b}_1\hat{\pi}_4 \qquad \hat{a}_2 = \hat{\pi}_1 - \hat{b}_2\hat{\pi}_4$$

Since $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, $\hat{b}_2$, $\hat{c}_1$, $\hat{c}_2$ are all single-valued functions of the $\hat{\pi}$, they are consistent estimates of the corresponding structural parameters. As mentioned earlier, this method is known as the *indirect least squares method*.

There is a simple counting rule available in the linear systems that we have been considering. This counting rule is also known as the *order condition* for identification. This rule is as follows: Let g be the number of endogenous variables in the system and k the total number of variables (endogenous and exogenous) missing from the equation under consideration. Then:

1. If $k = g - 1$, the equation is exactly identified.
2. If $k > g - 1$, the equation is over-identified.
3. If $k < g - 1$, the equation is under-identified.