AFRICA UNIVERSITY

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"Investing in Africa's future"

COLLEGE OF BUSINESS, PEACE ,LEADERSHIP AND GOVERNANCE

DISCRETE MATHEMATICS – CSC 201

END OF FIRST SEMESTER EXAMINATIONS
NOVEMBER 2019
LECTURER: Mr. Weston Govere
**DURATION: 3 HOURS**

INSTRUCTIONS

Answer **ANY FIVE** questions. Each question carry **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the part question.

Question 1

a) Show that $\left(p \wedge (\neg(\neg p \vee q))\right) \vee (p \wedge q) \equiv p$ [7]

b) Using Boolean algebra simplify the statement $\neg(r \to s) \to (\neg r)$ [6]

c) Compute the inverse, converse and contrapositive of the given statement *"If he works hard then he will pass the examination."* [3]

d) Trace the circuit from left to right .Write down the output of each logic gates. [4]
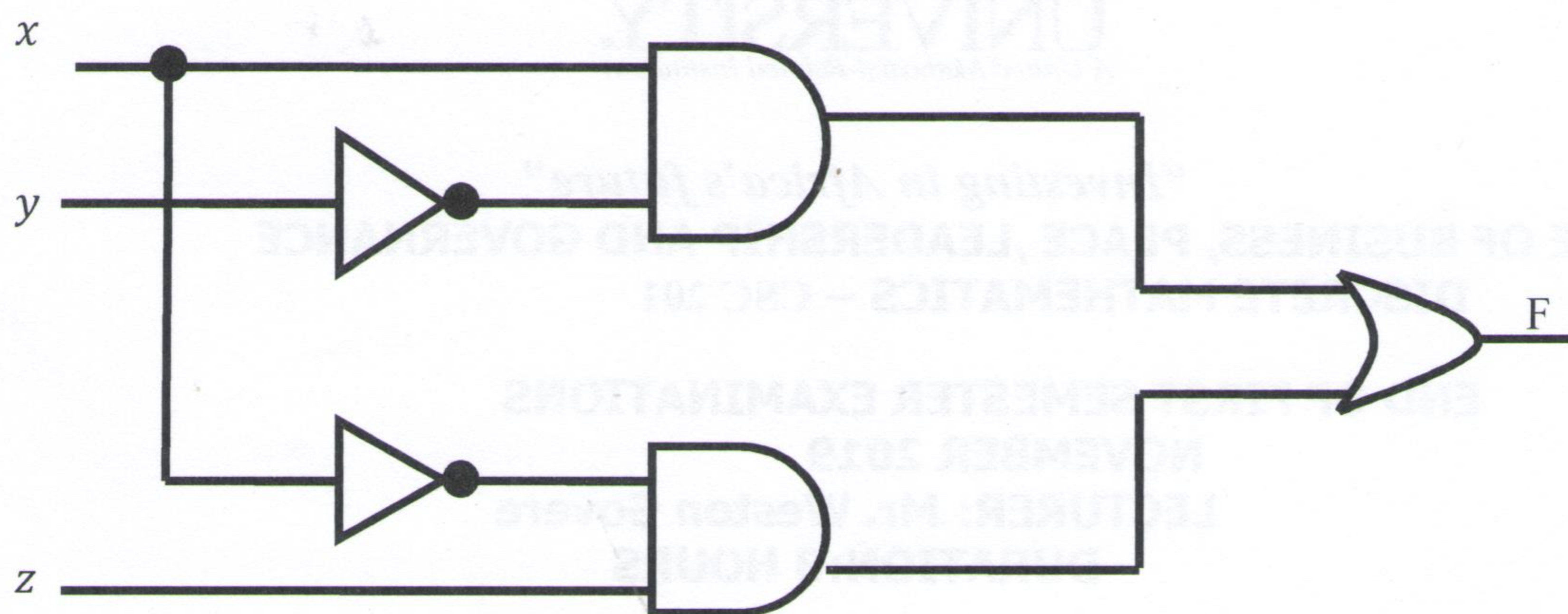
Question 2

a) Let R be a relation on a set A. define the following:

  i) R is reflective [2]

  ii) R is symmetric [2]

  iii) R is transitive [2]

  iv) R is an equivalence relation [2]

b) Given

$$A=\{1,2,3,4\}$$

Consider the following relation

$$R=\{(1,1),(1,2),(2,1),(2,2),(2,3),(2,4),(3,2),(3,3),(4,2)\}$$

  i. Draw its directed graph. [3]

  ii. Find the inverse of given relation [3]

b) Define a relation A on the set of real numbers R be defined as follows

$$\forall\, a, b \in R, a\, A\, b \Leftrightarrow a < b$$

i) Is A reflexive? **[2]**

ii) Is A symmetric? **[2]**

iii) Is A transitive? **[2]**

Question 3

a) Use the Euclidean algorithm to find gcd $(190, 34)$. **[3]**

b) Find integers s and t satisfying that gcd $(190, 34) = s(190) + t(34)$. **[3]**

c) Find the 7th term of the following geometric sequence

   $6, 18, 54\ldots$ **[5]**

d) Let $B = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$. Prove by the Principle of Mathematical Induction that

$$B = \begin{bmatrix} 1 & 4^n - 1 \\ 0 & 4^n \end{bmatrix} \text{ for } n = 1, 2, 3, \ldots$$ **[6]**
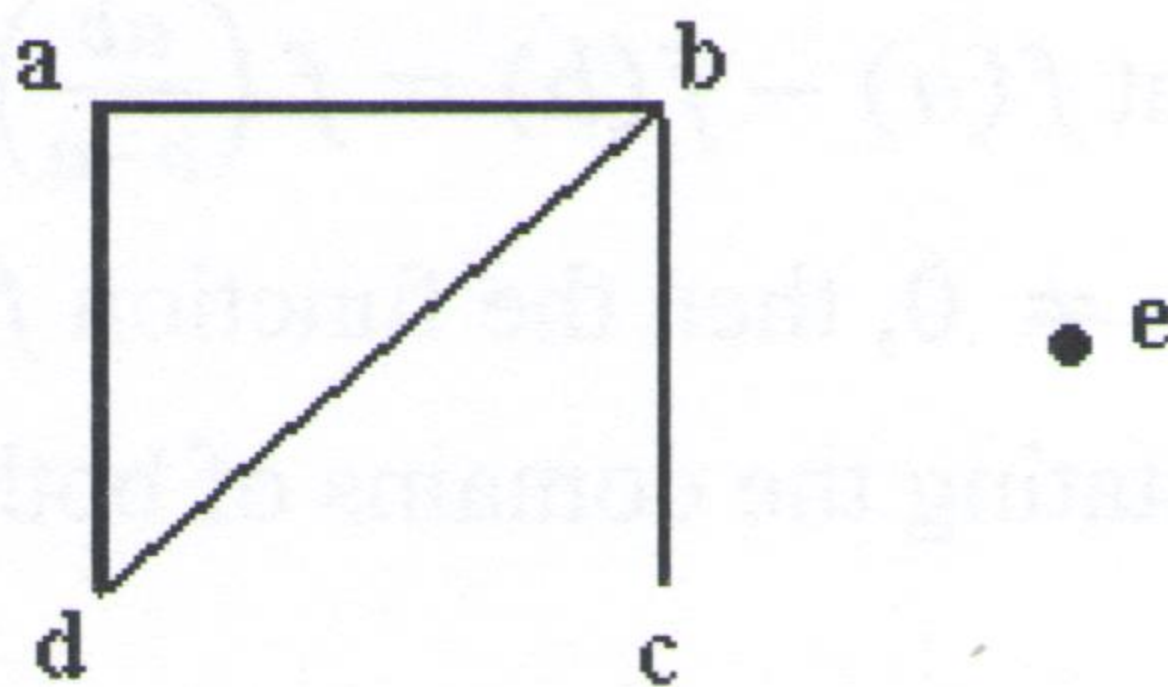
e) Prove by contradiction method, the statement:" if a and b are odd integers
   then $a + b$ is an even integer. **[3]**

Question 4

a) Prove that in a graph, the number of vertices with an odd degree is even.

   [3]

b) Determine whether the graph G and G' given below are isomorphic.

   [3]
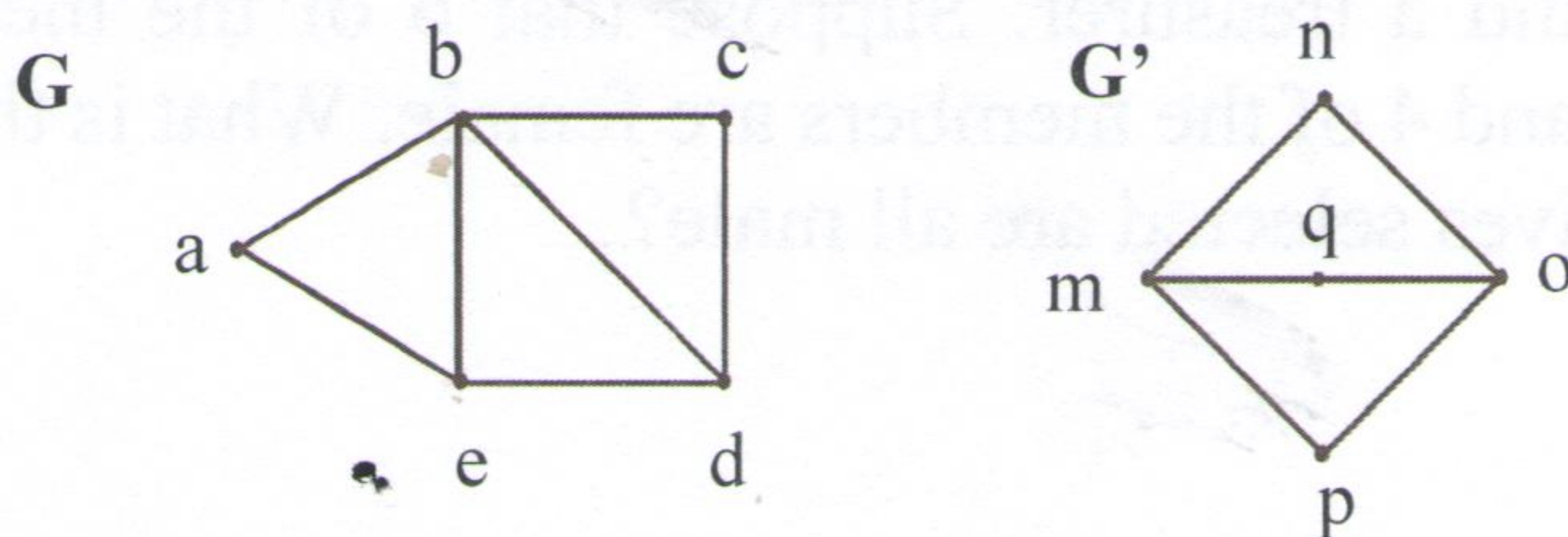
c) Find all non-isomorphic trees with four vertices. **[4]**

d) Find the degree sequence of the following graph **[5]**

e) Suppose that a connected planner simple graph has 25 edges. If a plane drawing of this graph has 10 faces, how many vertices does this graph have? **[5]**

Question 5

a) Mutare and Bulawayo are two of the cities competing for the National university games. (There are also many others). The organizers are narrowing the competition to the final 5 cities. There is a 20% chance that Mutare will be amongst the final 5. There is a 35% chance that Bulawayo will be amongst the final 5 and an 8% chance that both Mutare and Bulawayo will be amongst the final 5. What is the probability that Mutare or Bulawayo will be amongst the final 5. **[4]**

b) A bag contains 14 identical balls, 4 of which are red, 5 black and 5 white. Six balls are drawn from the bag. Find the probability that
  (i)   3 are red **[3]**
  (ii)   At least two are white. **[4]**

c) Here you choose 6 numbers from the integers 1, 2, 3, …, 47, 48, 49. Six winning numbers are chosen together with a bonus number. How many choices for the 6 winning numbers. **[3]**

d) We have a committee of $n = 10$ people and we want to choose a chairperson, a vice-chairperson and a treasurer. Suppose that 6 of the members of the committee are male and 4 of the members are female. What is the probability that the three executives selected are all male? **[6]**

Question 6

a) Find fog, gof where
$$f(x) = 2x + 1 \text{ and } g(x) = \frac{3}{x-1} , x \neq 1$$
[5]

b) If $f(x) = \frac{1}{x}$, show that $f(a) - f(b) = f\left(\frac{ab}{b-a}\right)$. **[3]**

c) Show that if $ad - bc \neq 0$, then the function $f(x) = \frac{ax+b}{cx+d}$ is one-to-one and find its inverse, stating the domains of both the function and its inverse. **[8]**

d) Find the domain and range of the following function given by

$$f(x) = \frac{\sqrt{(3x-5)(x+4)}}{x^3 - 16x}.$$ [4]

END OF EXAMINATION PAPER