AFRICA UNIVERSITY

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"Investing in Africa's future"

COLLEGE OF BUSINESS, PEACE, LEADERSHIP AND GOVERNANCE (CBPLG)

CALCULUS –NCSC 103

END OF FIRST SEMESTER EXAMINATIONS

NOVEMBER 2019

LECTURER: Mr.Timothy Makambwa

DURATION: 3 HOURS

INSTRUCTIONS

Answer **ALL** the questions in **Section A** and any **Three questions from Section B** and each question has **20** marks. Total possible mark is **100.**

Start **each** question on a new page on your answer sheet.

The marks allocated to **each** question are shown at the end of the section.

Section A (40 Marks)

Question One

Compute the following limits. Each limit is worth 5 marks. **Note** remember to simplify your answers.

a) $\lim_{x \to -2} \frac{x^2+3x+2}{x+2}$

b) $\lim_{n \to \infty} \frac{1+2+3+....n}{n^2}$

c) $\lim_{x \to} \frac{1}{x-1} - \frac{2}{x^2-1}$

d) $\lim_{x \to 0^+} \frac{\sin(6x)}{x}$

e) $\lim_{h \to 0} \frac{(1+h)^2-1}{h}$ [25]

Question Two

Differentiate.

a) $f(x) = e^x + x^3 - 5\ln x$

b) $g(q) = 8q^5 + 7q^4 - \frac{4}{7q^3}$

c) $y = \sqrt[5]{x} - \frac{2}{x} + 4e^x$

d. Find $\frac{d^3y}{dx^3}$ for $y = 2x^5 + 3x^3 + 9x - 5e^x$

[15]

Section B

Answer any three questions from this Section

Question Three

a) $x^2 + 2xy + y = 4$ Find the slope at (1,1)

b) Find the equation(s) of all horizontal tangent(s) to the graph of $y = x^3 - 3x - 4$. Show all work.

c) Find all of the following information and draw a graph of $y = x^3 - 12x + 3$

i) Increasing: ___________________________

ii) Decreasing: ___________________________

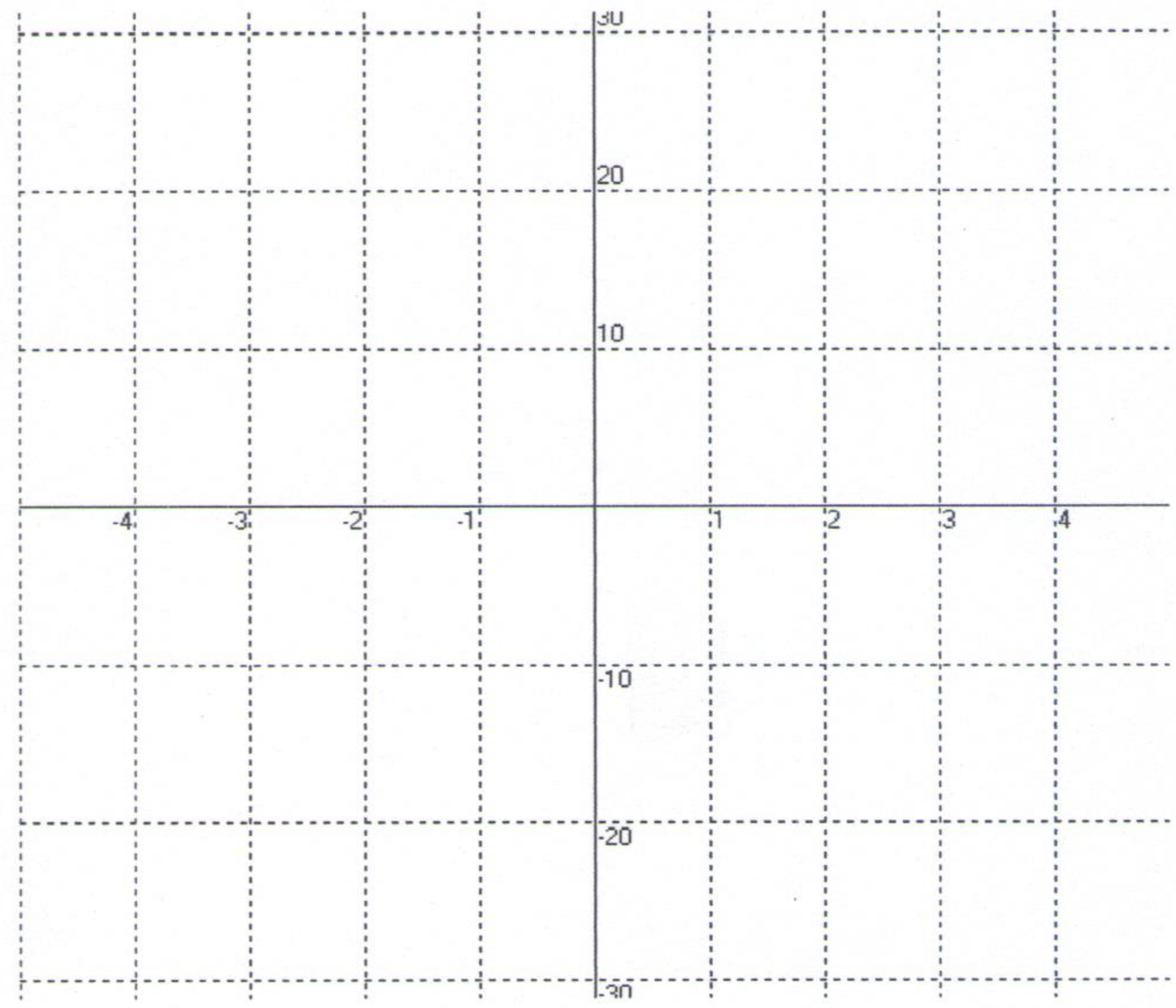
iii) Concave Up: ______________________

iv) Concave Down: ____________________

v) Maxima: __________________________

vi) Minima: __________________________

vii) Points of Inflection: ______________



Question Four

Integrate the following by making the substitution given

a) $\int e^{-x}dx$, $u=-x$ [3]
b) $\int x^3 (2+x^4)^5dx$, $u= 2+x^4$ [4]
c) $\int x^2 \sqrt{(x^3+1)}\, dx$, $u= x^3+1$ [4]
d) $\int \cos^3\theta \sin\theta \, d\theta$, $u= \cos\theta$ [4]
e) $\int \frac{\sec^2(1/x)}{x^2} dx$, $u= 1/x$ [5]

Question Five

a) An oil storage tank ruptures at time t= 0 and oil leaks from the tank at a rate of $r(t)= 100 e^{-0..01t}$ litres per minute. How much oil leaks out during the first hour?
b) A bacteria population starts with 400 bacteria and grows at a rate of $r(t)= (450.628)\, e^{1.12567t}$ bacteria per hour. How many bacteria will there be after three hours?
c) Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function $f(t)= 0.5 \sin(2\pi t/5)$has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t.

[5+7+8]

Question Six

a) Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$.
b) Determine the area of the region bounded by $y = 2x^2+10$ and $y = 4x +16$.
c) Determine the area of the region bounded by $y = 2x^2 +10$, $y = 4x +16$, $x = -2$ and $x = 5$
d) Determine the area of the region bounded by $x = -y^2 +10$ and $x =(y - 2)^2$.

[5+5+5+5]

Question Seven

a) Find the equation of the tangent and normal to the demand curve $y = 10\text{-}3x^2$ at (1, 7).

b) Find the slope of the tangent line at the point (0, 5) of the curve $y = ⅓\,(x^2 + 10x - 15)$. At what point of the curve the slope of the tangent line is 5 /8 ?

c) Determine the coefficients a and b so that the curve $y = ax^2 - 6x + b$ may pass through the point (0, 2) and have its tangent parallel to the x-axis at $x = 1.5$.

d) Determine the values of l and m so that the curve, $y = lx^2 + 3x + m$ may pass through the point (0, 1) and have its tangent parallel to the x-axis at x = 0.75.

[5+5+5+5]

Question Eight

a) Investigate the maxima and minima of the function $2x^3 + 3x^2 - 36x + 10$.

b) Find the absolute (global) maximum and minimum values of the function $f(x) = 3x^5 - 25x^3 + 60x + 1$ in the interval [-2, 1]

c) Find the points of inflection of the curve $y = 2x^4 - 4x^3 + 3$.

[6+8+6]

Mathematical Formulae

Integration Techniques

Many integration formulas can be derived directly from their corresponding derivative formulas, while other integration problems require more work. Some that require more work are substitution and change of variables, integration by parts, trigonometric integrals, and trigonometric substitutions.

Basic formulas

Most of the following basic formulas directly follow the differentiation rules.

1. $\int kf(x)\,dx = k\int f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc x \cot x\,dx = -\csc x + C$
11. $\int e^x\,dx = e^x + C$
12. $\int a^x\,dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{dx}{x} = \ln|x| + C$
14. $\int \tan x\,dx = -\ln|\cos x| + C$
15. $\int \cot x\,dx = \ln|\sin x| + C$
16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
17. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{x}{a} + C$

Applications of Integration

$\therefore$ Producer Surplus = $p_0q_0 - \int gx\,dx$ with limits from 0 to q_0

Consumer Surplus = $\int gx\,dx - p_0q_0$ with limits from 0 to q_0

Total Cost = $\int MCdx$

Total Revenue = $\int MRdx$

Differentiation

The chain rule

If *y* is a function of *u*, which is itself a function of *x*, then

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

differentiate the outer function and multiply by the derivative of the inner function

The product rule

If $y = uv$ then = $\frac{dy}{dx} = u.\frac{dv}{dx}$ x $v.\frac{du}{dx}$

This rule tells you how to differentiate the product of two functions: **multiply each function by the derivative of the other and add**

The quotient rule

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v.du/dx - u.dv/dx}{v^2}$

This rule tells you how to differentiate the quotient of two functions: **bottom times derivative of top, minus top times derivative of bottom, all over bottom squared**